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On active vibration isolation of floating raft system

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Abstract

A novel analytical active–passive model of floating rafts, a type of special isolation structures which present high-level vibration isolation and are widely used in large ships and submarines particularly, is developed for the first time. Then the mobility matrices of the subsystems are derived, thereby a general mathematical description of this combined active–passive model is realized. Based on the model, the concepts and relationships of machine control, raft control and full control are extensively discussed. The solution of the power flow transmitted into the foundation is obtained, and power transmission characteristics of the system are investigated under different control types when minimization of total power flow strategy is applied. Through numerical simulations, the control efficiencies of the different control types (machine control, raft control and full control) are compared, illustrating the efficiency of the presented model, obtaining some valuable results, and presenting some general design principles of the active floating raft isolation systems.

1. Introduction

With the development of the vibration control techniques and increasing strict requirements for the vibration isolation in industry and everyday life, classical one-stage isolation systems exhibit poor performances [1–4]. To achieve more efficient vibration cancellation, some two-stage even multi-stage isolation systems have received increasingly research attention in recent years [5–8],

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and active control techniques of two-stage isolation systems are even discussed [7,8]. The results illustrate that two-stage isolation systems can achieve better vibration cancellation than one-stage isolation systems, especially at high frequencies [5,9]. Based on two-stage isolation systems, floating raft systems, a type of special vibration isolation structures, were developed about twenty years ago especially for ships and submarines. They can isolate vibration of hosts and auxiliary machines and reduce the structural noise of ships and submarines effectively. They can also protect the equipments and instruments in ships and submarines from being damaged, and makes them to be operated properly when ships and submarines are subjected to external loads and sudden shocks. Because isolation of floating raft structures is a key technique for ships, in particular submarines, it has drawn much attention in recent years [10–17]. However, according to the recent literatures, all the works [10–17] about floating raft isolation systems are only limited to passive systems.

Active control techniques are able to dynamically adapt the characteristic parameters of the systems or structures in order to meet the strict requirements of vibration isolation. Because of their great adaptive capacity, much attention has been devoted to active isolation systems. Substantial works on active vibration control have been published where the power flow transmitted to flexible foundations or receivers has been considered as the cost function to maximizing the cancellation of vibration [18–23]. However, in these researches, the theoretical models of the isolation systems are mostly of one-stage, and more complicated two-stage active isolation systems, such as floating raft systems, have not been dealt with.

Based on the perspective above, a novel analytical model is developed to describe active floating raft isolation systems. Some active actuators are inserted between machines and an intermediate raft as well as the intermediate raft and the foundation. The active actuators are installed parallel to the passive isolators. The mobility or impedance matrix technique is used to derive the mobility matrices of the subsystems, respectively [24], such as machines, mounts, floating raft, and plate foundation. With the general mobility matrices of the subsystems, the passive and active systems are united into one combined system which can be changed to a passive system if the active forces are set to zero or the actuators are removed from the combined system. The transmission characteristics of the power flow in the active floating raft isolation system are investigated in the proposed model, and some essential design principles of the active floating raft isolation system are presented here. The results will provide important instructions for vibration isolation design of active floating raft systems.

2. Analytical model and dynamic analysis of floating raft system

As an advanced isolation system, the floating raft isolation system is more complicated than most general two-stage isolation systems and it provides much better vibration reduction than the latter. Fig. 1 shows an analytical model of active floating raft isolation system. Two or more machines are mounted on a single intermediate raft structure. The overall isolation system can be divided into five subsystems: (i) machines A, (ii) upper mount system B including passive isolators and active actuators, (iii) intermediate raft R, (iv) lower mount system D including passive isolators and actuators, and (v) foundation C. The intermediate raft structure is considered as a rigid block and the flexible foundation is modeled as a thin

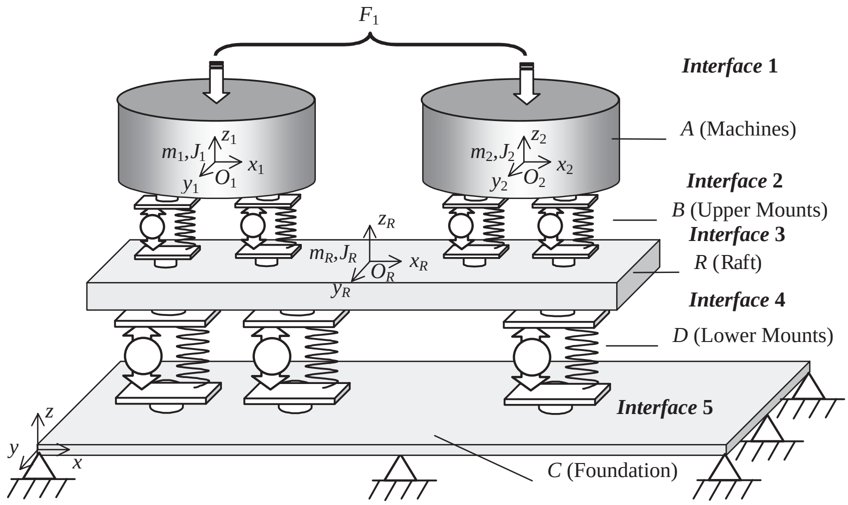

Fig. 1. Analytical model of an active floating raft isolation system.

rectangular plate simply supported at its four edges. These five subsystems are connected at a finite number of junctions by the mounts. In engineering, the vertical vibration energy is more significant than that of other directions especially in low-frequency band, so only the vertical forces and the resulting motions of the system are concerned in the presented model. Here both upper and lower mount systems are combined structures where the active actuators are parallel with the passive isolators. The passive isolators are modeled as damped springs and the action of the active actuators can be described as a pair of axial reactive forces acting at the top and bottom ends of the mounts attached to the machines, the intermediate raft structure and the flexible foundation.

2.1. Dynamic analysis of the machines

The dynamics of the active control system is studied using a mobility matrix technique. The governing equation of mobility matrix can be expressed as

$$\begin{Bmatrix} V_{At} \\ V_{Ab} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} F_{At} \\ F_{Ab} \end{Bmatrix} \tag{1}$$

where $F_{At}, F_{Ab}, V_{At}, V_{Ab}$ are, respectively, the upper and the lower forces and their corresponding velocities of the subsystem A, the abbreviation of *bottom*, b, denotes the bottom output, and the abbreviation of *top*, t, indicates the top output of the corresponding subsystem. The forces and the resulting velocities can be written as

$$V_{At} = \{V_{A_1t}, V_{A_2t}\}^{\mathrm{T}} = \{V_{o1}, V_{o2}\}^{\mathrm{T}}, \qquad F_{At} = \{F_{A_1t}, F_{A_2t}\}^{\mathrm{T}} = \{F_{o1}, F_{o2}\}^{\mathrm{T}} \tag{2,3}$$

$$V_{Ab} = \{V_{A_11b}, V_{A_12b}, V_{A_21b}, V_{A_22b}\}^{\mathrm{T}}, \qquad F_{Ab} = \{F_{A_11b}, F_{A_12b}, F_{A_21b}, F_{A_22b}\}^{\mathrm{T}} \tag{4,5}$$

where $F_{A_rt} = F_{or}, V_{A_rt} = V_{or}, F_{A_rsb}, V_{A_rsb}$ $(r, s = 1, 2)$ are, respectively, the forces and their corresponding velocities at the top and bottom ends of the sth mount attached to the rth machine of subsystem A. The governing equation of subsystem A can be rewritten in full as

$$\begin{Bmatrix} V_{A_1t} \\ V_{A_2t} \\ V_{A_11b} \\ V_{A_12b} \\ V_{A_21b} \\ V_{A_22b} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} F_{A_1t} \\ F_{A_2t} \\ F_{A_11b} \\ F_{A_12b} \\ F_{A_21b} \\ F_{A_22b} \end{Bmatrix} = \begin{bmatrix} \begin{bmatrix} A_{11}^{(1)} & 0 \\ 0 & A_{11}^{(2)} \end{bmatrix} & \begin{bmatrix} A_{12}^{(1)} & 0 \\ 0 & A_{12}^{(2)} \end{bmatrix} \\ -\begin{bmatrix} A_{21}^{(1)} & 0 \\ 0 & A_{21}^{(2)} \end{bmatrix} & \begin{bmatrix} A_{22}^{(1)} & 0 \\ 0 & A_{22}^{(2)} \end{bmatrix} \end{bmatrix} \begin{Bmatrix} F_{A_1t} \\ F_{A_2t} \\ F_{A_11b} \\ F_{A_12b} \\ F_{A_21b} \\ F_{A_22b} \end{Bmatrix} \tag{6}$$

where the elements can be given, respectively, as

$$A_{11}^{(r)} = \frac{1}{\overline{M}_r} \tag{7}$$

$$A_{12}^{(r)} = \left[\frac{1}{\overline{M}_r}, \quad \frac{1}{\overline{M}_r} \right] \tag{8}$$

$$A_{22}^{(r)} = \begin{bmatrix} -\dfrac{1}{\overline{M}_r} - \dfrac{x_{r1}^2}{\overline{J}_r} & -\dfrac{1}{\overline{M}_r} - \dfrac{x_{r2}x_{r1}}{\overline{J}_r} \\ -\dfrac{1}{\overline{M}_r} - \dfrac{x_{r2}x_{r1}}{\overline{J}_r} & -\dfrac{1}{\overline{M}_r} - \dfrac{x_{r2}^2}{\overline{J}_r} \end{bmatrix} \tag{9}$$

in which $\overline{M}_r = \mathrm{j}\omega m_r, \overline{J}_r = \mathrm{j}\omega J_r$, m_r, J_r are, respectively, the mass and the moment of inertia of the rth machine, x_{rs} $(r, s = 1, 2)$ is the local coordinate of junction of the sth mount attached to the rth machine, and $\mathrm{j} = \sqrt{-1}$.

2.2. Dynamic analysis of the floating raft system

As mentioned above, the intermediate raft structure is considered as a rigid rectangular block in the presented model. According to the dynamics theory of rigid bodies, the mobility matrix equation of the intermediate raft structure can be easily obtained as

$$\begin{Bmatrix} V_{Rt} \\ V_{Rb} \end{Bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{Bmatrix} F_{Rt} \\ F_{Rb} \end{Bmatrix} \tag{10}$$

The equation above can be further expressed as

$$\begin{Bmatrix} V_{R1t} \\ \vdots \\ V_{R4t} \\ V_{R1b} \\ V_{R2b} \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{\overline{M}_R}+\dfrac{x_{R1t}^2}{\overline{J}_R} & \cdots & \dfrac{1}{\overline{M}_R}+\dfrac{x_{R4t}x_{R1t}}{\overline{J}_R} & -\dfrac{1}{\overline{M}_R}-\dfrac{x_{R1t}x_{R1b}}{\overline{J}_R} & -\dfrac{1}{\overline{M}_R}-\dfrac{x_{R1t}x_{R2b}}{\overline{J}_R} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \dfrac{1}{\overline{M}_R}+\dfrac{x_{R1t}x_{R4t}}{\overline{J}_R} & \cdots & \dfrac{1}{\overline{M}_R}+\dfrac{x_{R4t}^2}{\overline{J}_R} & -\dfrac{1}{\overline{M}_R}-\dfrac{x_{R4t}x_{R1b}}{\overline{J}_R} & -\dfrac{1}{\overline{M}_R}-\dfrac{x_{R4t}x_{R2b}}{\overline{J}_R} \\ \dfrac{1}{\overline{M}_R}+\dfrac{x_{R1t}x_{R1b}}{\overline{J}_R} & \cdots & \dfrac{1}{\overline{M}_R}+\dfrac{x_{R4t}x_{R1b}}{\overline{J}_R} & -\dfrac{1}{\overline{M}_R}-\dfrac{x_{R1b}^2}{\overline{J}_R} & -\dfrac{1}{\overline{M}_R}-\dfrac{x_{R1b}x_{R2b}}{\overline{J}_R} \\ \dfrac{1}{\overline{M}_R}+\dfrac{x_{R1t}x_{R2b}}{\overline{J}_R} & \cdots & \dfrac{1}{\overline{M}_R}+\dfrac{x_{R4t}x_{R2b}}{\overline{J}_R} & -\dfrac{1}{\overline{M}_R}-\dfrac{x_{R1b}x_{R2b}}{\overline{J}_R} & -\dfrac{1}{\overline{M}_R}-\dfrac{x_{R2b}^2}{\overline{J}_R} \end{bmatrix} \begin{Bmatrix} F_{R1t} \\ \vdots \\ F_{R4t} \\ F_{R1b} \\ F_{R2b} \end{Bmatrix} \tag{11}$$

where $F_{Rkt}, V_{Rkt}, F_{Rlb}, V_{Rlb}$ ($k = 1,2,3,4;\ l = 1,2$) are the forces and their corresponding velocities at the top and bottom ends of the kth upper mount and the lth lower mount attached to the floating raft subsystem R at the local floating raft coordinates x_{Rkt}, x_{Rlb} ($k = 1,2,3,4;\ l = 1,2$), respectively, $\overline{M}_R = \mathrm{j}\omega m_R, \overline{J}_r = \mathrm{j}\omega J_R$, and m_R, J_R are, respectively, the mass and the moment of inertia of the floating raft subsystem R.

2.3. Dynamic analysis of the upper and lower passive–active mounts

For the upper and lower mount systems, the weights of the isolators and actuators can be neglected because they are so small comparing with that of other subsystems such as machines, intermediate raft structure and foundation. The omission will not affect the dynamic characteristics or change the vibration behaviors of the overall isolation system significantly. Hence, the complex stiffness matrices of the upper and lower mount systems can be described as

$$K_B = \begin{bmatrix} k_{B1}^* & & & \\ & k_{B2}^* & & \\ & & \ddots & \\ & & & k_{Bk}^* \end{bmatrix}, \quad K_D = \begin{bmatrix} k_{D1}^* & & & \\ & k_{D2}^* & & \\ & & \ddots & \\ & & & k_{Dl}^* \end{bmatrix} \tag{12,13}$$

where $k_{Bk}^* = k_{Bk}(1 + \mathrm{j}\eta_{Bk}), k_{Dl}^* = k_{Dl}(1 + \mathrm{j}\eta_{Dl})$ ($k = 1,2,3,4; l = 1,2$) are the complex stiffness of the kth upper isolator and the lth lower isolator, and η_{Bk}, η_{Dl} are the corresponding damping loss factors, respectively.

As mentioned above, the action of the active actuators is modeled as a pair of reacting axial forces acting at the top and the bottom points of the mounts. Since the active actuators are parallel to the passive isolators, the dynamic equation of the upper and lower mounts can be expressed in the following forms:

$$F_{Bt} = \overline{K}_B(V_{Bt} - V_{Bb}) + U_B \tag{14}$$

$$F_{Dt} = \overline{K}_D(V_{Dt} - V_{Db}) + U_D \tag{15}$$

where

$$\overline{K}_B = \frac{K_B}{j\omega}, \quad \overline{K}_D = \frac{K_D}{j\omega} \tag{16,17}$$

and

$$V_{Bt} = \{V_{B1t}, \ldots, V_{B4t}\}^T, \quad V_{Bb} = \{V_{B1b}, \ldots, V_{B4b}\}^T, \quad F_{Bt} = \{F_{B1t}, \ldots, F_{B4t}\}^T \tag{18−20}$$

$$V_{Dt} = \{V_{D1t}, V_{D2t}\}^T, \quad V_{Db} = \{V_{D1b}, V_{D2b}\}^T, \quad F_{Dt} = \{F_{D1t}, F_{D2t}\}^T \tag{21−23}$$

in which $V_{Bkt}, V_{Bkb}, V_{Dlt}, V_{Dlb}$ ($k = 1, 2, 3, 4;\ l = 1, 2$) are velocities of the top and bottom ends of the kth upper mount and the lth lower mount, and F_{Bkt}, F_{Dlt} ($k = 1, 2, 3, 4;\ l = 1, 2$) are the corresponding forces, respectively.

2.4. Dynamic analysis of the flexible plate foundation

In a similar way, the governing equation for the rectangular plate foundation by the mobility matrix, on which the m lower mounts are mounted, can be obtained as

$$\begin{Bmatrix} V_{C1} \\ V_{C2} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{Bmatrix} F_{C1} \\ F_{C2} \end{Bmatrix} \tag{24}$$

or more concisely

$$V_c = CF_c \tag{25}$$

where F_{Cl}, V_{Cl} ($l = 1, 2$) are the force and its corresponding velocity transmitted into the foundation subsystem C, respectively, and $C_{m\times n}$ is the mobility matrix of the plate foundation. The latter is a square matrix with $m \times m$ dimensions.

The transfer mobility from the point $\sigma_p = (x_p, y_p)$ to the point $\sigma_q = (x_q, y_q)$ can be determined by

$$C_{pq} = \frac{j\omega}{M_b} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\psi_{mn}(\sigma_p)\psi_{mn}(\sigma_q)}{\omega_{mn}^2(1 + j\delta) - \omega^2} \tag{26}$$

where

$$\psi_{mn}(\sigma) = \sin\left(\frac{m\pi x}{l_x}\right) \sin\left(\frac{m\pi y}{l_y}\right) \tag{27}$$

is the modal shape function of the rectangular thin plate simply supported at its four edges, δ is the damping factor of the plate foundation, l_x, l_y are the length and the width of the plate foundation, respectively, and ω_{mn} is the natural frequencies of the rectangular thin plate simply supported at its four edges [25].

3. System combination and solution of power flows

Thus, the dynamic analysis of each subsystem has been conducted. The following attempt is to assemble all the subsystems to an overall system by the mobility or impedance technique. The total power flow transmitted into the foundation, being considered as a cost function, is solved for employing an improved power flow minimization strategy to achieve reducing of the vibration of the system.

3.1. System combination

From Fig. 1, the relationships of the transmitted forces and the corresponding velocities on the interfaces of the subsystems can be easily determined as

$$\begin{aligned} &F_{Ab} = F_{Bt}, \quad V_{Ab} = V_{Bt}, \quad F_{Bb} = F_{Rt}, \quad V_{Bb} = V_{Rt} \\ &F_{Rb} = F_{Dt}, \quad V_{Rb} = V_{Dt}, \quad F_{Db} = F_C, \quad V_{Db} = V_C \end{aligned} \tag{28−35}$$

As the weights of the passive isolators and the active actuators are neglected, the following equations on the ends of the upper and lower mounts can be obtained,

$$F_{Bb} = F_{Bt}, F_{Db} = F_{Dt} \tag{36}$$

By synthesizing the equations above, the transmitted forces and corresponding velocities on each of the interfaces of the subsystems in the floating raft isolation system can be easily solved. In practical vibration isolation, the main aim of the vibration control is to isolate the vibration transmitting into the foundation, so the following section focuses on the solution of the power flow transmitted into the foundation. Once the force and resulting velocity of the flexible foundation are obtained, some control strategies can be employed to reduce vibration of the system. In this case, the power flow transmitted into the foundation acts as the cost function to be minimized. Therefore, one aims at solving the force and the resulting velocity transmitted into the foundation, and further deriving the cost function of the transmitted power flow.

The forces and the corresponding velocities transmitted into interface 5 (the foundation) are

$$F_C = T\overline{K}_D R_{21}(I - \overline{K}_B A_{22} + \overline{K}_B R_{11})^{-1}\overline{K}_B A_{21} F_1 + [T\overline{K}_D R_{21}(I - \overline{K}_B A_{22} + \overline{K}_B R_{11})^{-1}, T]U \tag{37}$$

$$V_C = CT\overline{K}_D R_{21}(I - \overline{K}_B A_{22} + \overline{K}_B R_{11})^{-1}\overline{K}_B A_{21} F_1 + C[T\overline{K}_D R_{21}(I - \overline{K}_B A_{22} + \overline{K}_B R_{11})^{-1}, T]U \tag{38}$$

where

$$T = [I + \overline{K}_D R_{21}(I - \overline{K}_B A_{22} + \overline{K}_B R_{11})^{-1}\overline{K}_B R_{12} - \overline{K}_D R_{22} + \overline{K}_D C]^{-1} \tag{39}$$

and

$$U = \{U_B, U_D\}^{\mathrm{T}} = \{u_{B1}, u_{B2}, \ldots, u_{B4}, u_{D1}, u_{D2}\}^{\mathrm{T}} \tag{40}$$

For achieving optimal control of power flow into the foundation, the force and velocity of the foundation shown in Eqs. (37) and (38) can be expressed as the summation of an active portion and a passive portion, as

$$F_C = T_1 U + T_2 F_1 \tag{41}$$

$$V_C = T_3 U + T_4 F_1 \tag{42}$$

where, the coefficients of the active and passive portions in Eqs. (41) and (42) are, respectively,

$$T_1 = [T\overline{K}_D R_{21}(I - \overline{K}_B A_{22} + \overline{K}_B R_{11})^{-1}, T] \tag{43}$$

$$T_2 = T\overline{K}_D R_{21}(I - \overline{K}_B A_{22} + \overline{K}_B R_{11})^{-1}\overline{K}_B A_{21} \tag{44}$$

$$T_3 = CT_1, \quad T_4 = CT_2 \tag{45,46}$$

Because the floating raft isolation system considered in this paper is of two-stage, there are three choices to install the active actuators: (1) Only installing the active actuators parallel to the passive isolators between machines and intermediate raft structure, the active forces of the actuators will affect the vibration behaviors of the machines directly. This type of control is termed as *machine control*. (2) Only installing the active actuators parallel to the passive isolators between intermediate raft structure and foundation, the active force acting on intermediate raft structure and foundation will have significant effects on the vibration behaviors of the intermediate raft structure. This type of control is named as *raft control*. (3) Concurrently installing the active actuators parallel to the passive isolators between the machines and the intermediate raft structure as well as between the intermediate raft structure and the foundation, the active forces will have contributions to the vibration characteristics of the overall isolation system. This type of control is called as *full control* of the floating raft isolation system.

With reference to the three types of control above, coefficients of the active portions in Eqs. (41) and (42) are different from each other. They illustrate the corresponding forces and velocities transmitted into the foundation in different types of control, respectively. In the operation of full control, a particular type of full control will be degenerated to machine control only when the first portions in the coefficient vectors T_1 and T_3 are adopted, respectively. Similarly, a particular type of full control will be degenerated to raft control only when the second portions in the coefficient vectors T_1 and T_3 are adopted, respectively. Therefore, the full control mode is the superposition of machine control and raft control. In addition, the active floating raft isolation system will be degenerated to the classical passive floating raft system if the control vectors U in Eqs. (41) and (42) are set to zero. The relationships of the three control types of the floating raft isolation system and the passive floating raft system are illustrated in Fig. 2, in which the evolution of the passive and active systems can be exhibited clearly.

3.2. An improved power flow minimization strategy

The transmission of total power flow into the foundation makes a comprehensive illustration of the vibration mechanism of the overall system. It illustrates the transmission characteristics of the vibration energy of the overall system in detail. It is feasible to formulate the transmission of total power flow into the foundation as a cost function in order to determine the optimum control

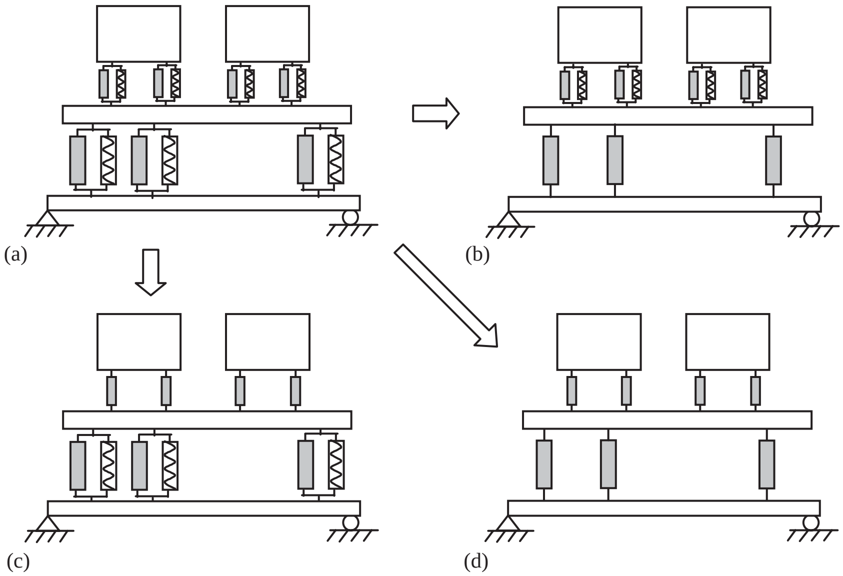

Fig. 2. Relationships of three types of control: (a) full control; (b) machine control; (c) raft control of the active floating raft system and (d) the passive floating raft system.

vector. As definition of power flow [5], the cost function of vibration reduction of the floating raft isolation system can be constructed as

$$P_{\text{trans}} = \tfrac{1}{2}\mathrm{Re}(F_C^{\mathrm{H}} V_C) = \tfrac{1}{4}(F_C^{\mathrm{H}} V_C + V_C^{\mathrm{H}} F_C) \tag{47}$$

where H is conjugate and transpose of the corresponding matrices or vectors.

Substituting the expressions in Eqs. (41) and (42) into Eq. (47), the cost function can be expressed as

$$J = \tfrac{1}{4}(F_C^{\mathrm{H}} V_C + V_C^{\mathrm{H}} F_C) = U^{\mathrm{H}} A U + U^{\mathrm{H}} b + b^{\mathrm{H}} U + c \tag{48}$$

where

$$A = T_1^{\mathrm{H}}(C + C^{\mathrm{H}})T_1/4, \qquad b = T_1^{\mathrm{H}}(C + C^{\mathrm{H}})T_2 F_1/4 \tag{49,50}$$

If matrix A in the above equation is positive-definite, Eq. (48) is in a standard Hermitain quadratic format and the existence of a minimum is guaranteed. The system, therefore, must have an optimum control vector in the form [26]

$$U_{\text{opt}} = -A^{-1} b \tag{51}$$

In practice, quantity of the optimum control vector depends on the maximum output of active actuators. The optimum control vector will be limited by this maximum output. Other than ensuring minimization of power flow, in general, the cost function should consider requirements of control vectors in the design of control systems. In other terms, the control system should have the actuators exert their potential adequately in order to better satisfy the aims of control. At the same time, the actuators should not exceed their output capacity. According to the factors above, the cost function in Eq. (48) can be rewritten as

$$\tilde{J} = \tfrac{1}{4}(F_C^{\mathrm{H}} V_C + V_C^{\mathrm{H}} F_C) + U^{\mathrm{H}} R U \tag{52}$$

Then, Eq. (49) is improved as

$$\tilde{A} = T_1^{\mathrm{H}}(C + C^{\mathrm{H}})T_1/4 + R \tag{53}$$

where R is the weight matrix of control vectors. In physics' viewpoint, a larger weight of the control vectors denotes a larger actuator output. On the contrary, a smaller weight suggests a lower output of the actuators. Instituting optimum force vector to Eq. (48), the transmission of total power flow into the foundation can be obtained easily.

4. Transmission characteristics and control of transmission of power flow

Power flow, a comprehensive performance index, can reveal the transmission mechanics of vibration in terms of energy. Obviously, minimizing the total power flow transmitted into the foundation can depress structural vibration and reduce acoustic radiation efficiently. To investigate the transmission characteristics of vibration in the floating raft isolation system, an improved power flow minimization strategy is employed in this paper. A numerical example is presented in which the parameters of machines and intermediate raft structure are $m_1 = m_2 = 2106.0\,\mathrm{kg}$, $J_1 = J_2 = 507.0\,\mathrm{kg\,m^2}$, $m_R = 2130\,\mathrm{kg}$, $J_R = 1822.0\,\mathrm{kg\,m^2}$, the characteristic parameters of passive mounts are $k_{Bk} = 3.24\,\mathrm{e}6\,\mathrm{N/m}$, $k_{Dl} = 2.39\mathrm{e}7\,\mathrm{N/m}, \eta_{Bk} = 0.1$, $\eta_{Dl} = 0.05$, the dimension, density, damping factor and Young's Modulus of plate foundation are $4\,\mathrm{m} \times 3\,\mathrm{m} \times 0.05\,\mathrm{m}$, $\rho = 7850\,\mathrm{kg/m^3}$, $\delta = 0.1$, $E = 210\,\mathrm{e}9\,\mathrm{N/m^2}$, respectively.

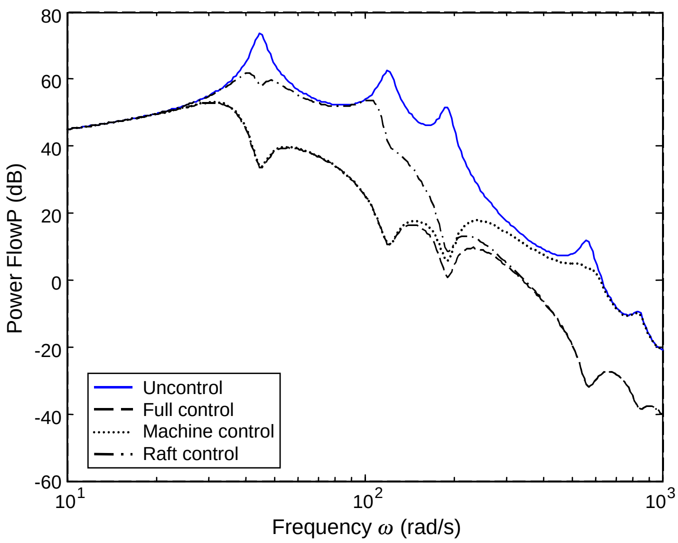

Fig. 3. Transmission of power flow into foundation when the minimization strategy of total power flow is employed.

Fig. 3 shows transmission of power flow into foundation for different control types with respect to the total power flow minimization strategy. In this figure, the power transmitted into the foundation by machine control, raft control and full control are compared among themselves. It is obvious that full control has a more excellent performance than the rests of the control types. At low frequencies, the plots of the power flow transmitted into the foundation by machine control and full control are almost identical. Machine control's capacity disappears gradually with increasing of frequency. On the other hand, raft control becomes more and more significant until a stage where the power plot of raft control agrees with that of full control. Furthermore, when the minimization strategy of total power flow is applied, the transmission of power flow into foundation reduces substantially. The plots of controlled power flow have some pronounced notches at the corresponding peaks of uncontrolled power, which forms an important characteristic of the control of power flow. Although machine control, raft control or full control can significantly depress the quantities of transmission of power flow into the foundation, full control is the most efficient one because its effect is equivalent to the superposition of machine control and raft control. As a result, it gains excellent vibration isolation and noise control in a broad frequency band.

Fig. 4 shows the transmission of power flow into machines for the three control types. In the figure, it can be concluded that all control types of power flow are able to reduce in different degrees the transmission of power flow into machines. Among them, the capacity of machine control is most significant while that of raft control is smallest. This is because the action of the lower actuators becomes weak through the intermediate raft structure and the upper mounts and therefore affects rarely the behaviors of machines in the raft control system. For machine control, the active forces of upper actuators substantially absorb the vibration energy of machines.

Fig. 5 illustrates the plots of transmission of power to every subsystem in the passive floating raft isolation system. The transmission of power to the subsystems when full control is employed

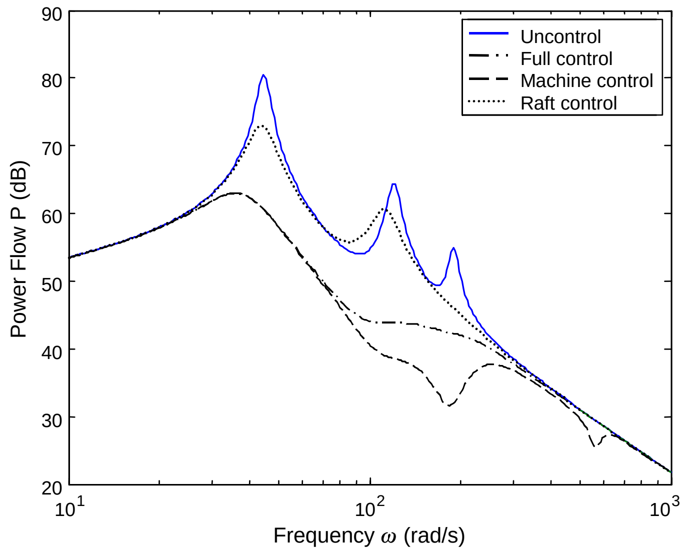

Fig. 4. Transmission of power flow into machines when the minimization strategy of total power flow is employed.

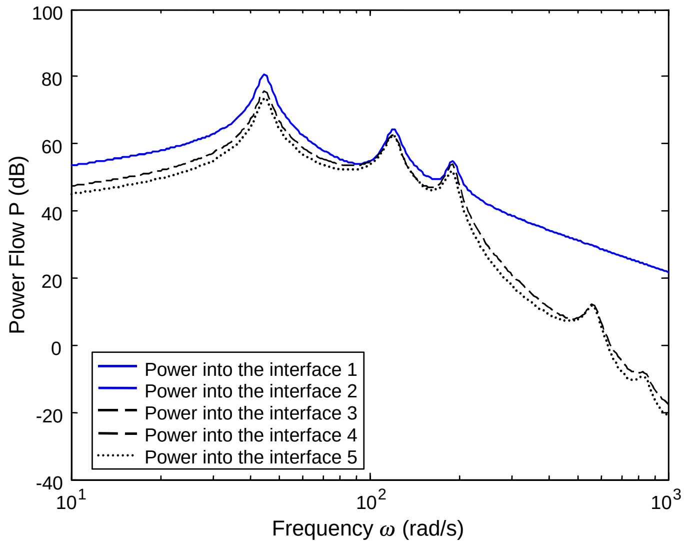

Fig. 5. Transmission of power flow to subsystems of passive floating raft isolation system.

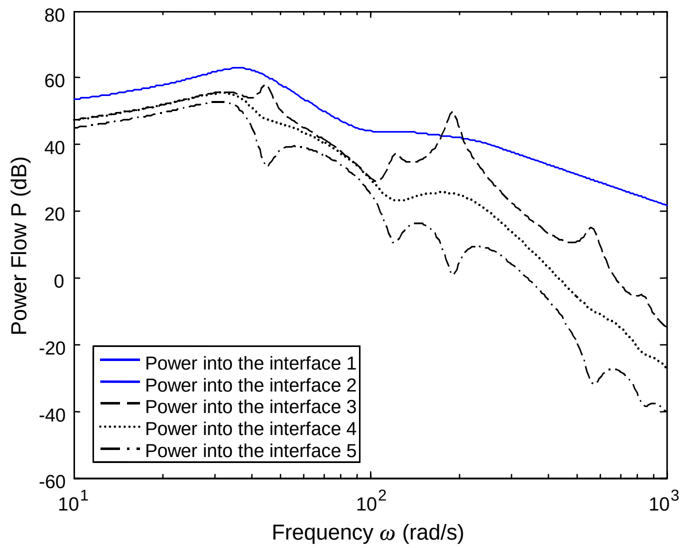

Fig. 6. Transmission of power flow to subsystems of active floating raft isolation system by full control.

is shown in Fig. 6. In the passive system, the transmission of power into interface 1 (i.e., the top of machines) almost equals to the transmission of power into interface 2 (i.e., the top of upper mounts). Similarly, the transmission of power into interface 3 (i.e., the top of intermediate raft structure) is almost the same as the transmission of power into interface 4 (i.e., the top of lower

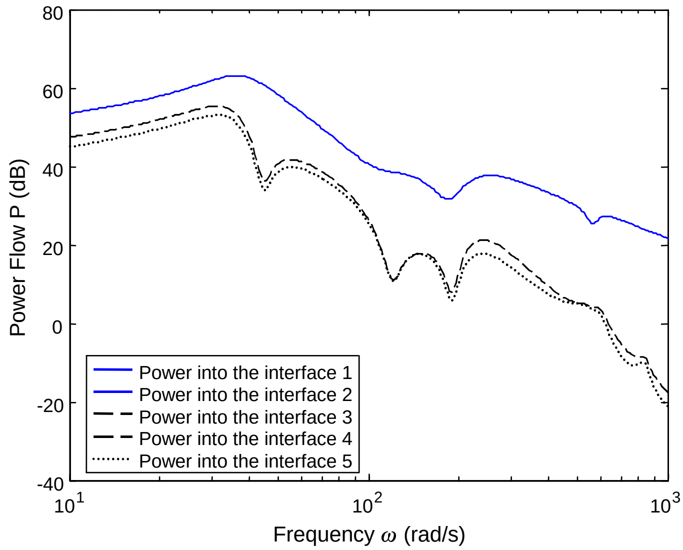

Fig. 7. Transmission of power flow to subsystems of active floating raft isolation system by machine control.

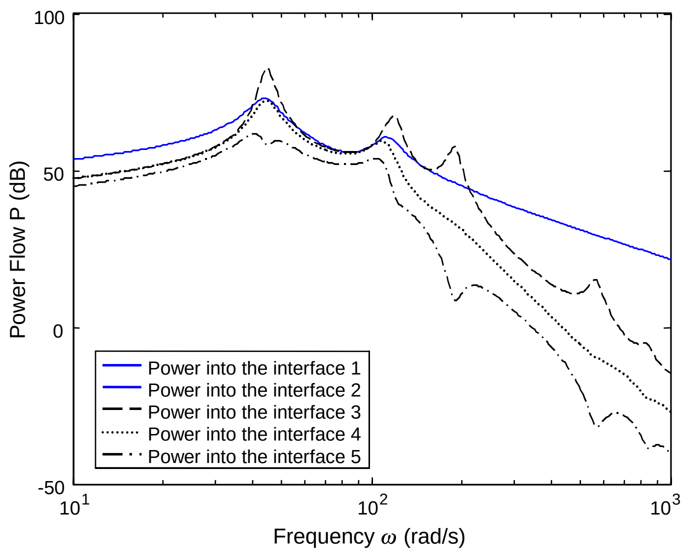

Fig. 8. Transmission of power flow to subsystems of active floating raft isolation system by raft control.

mounts). It is not difficult to understand that the machines and intermediate raft structure, both modeled as rigid bodies, have no mechanism of absorbing vibration energy in the passive isolation system. With reference to the active floating raft isolation system by full control, dynamic characteristics of the overall system are changed due to the action of actuators shown in Fig. 6. The power transmitted into interface 3 is significantly different from that transmitted into interface 4, especially in the mid- and high-frequency band above 100 rad/s.

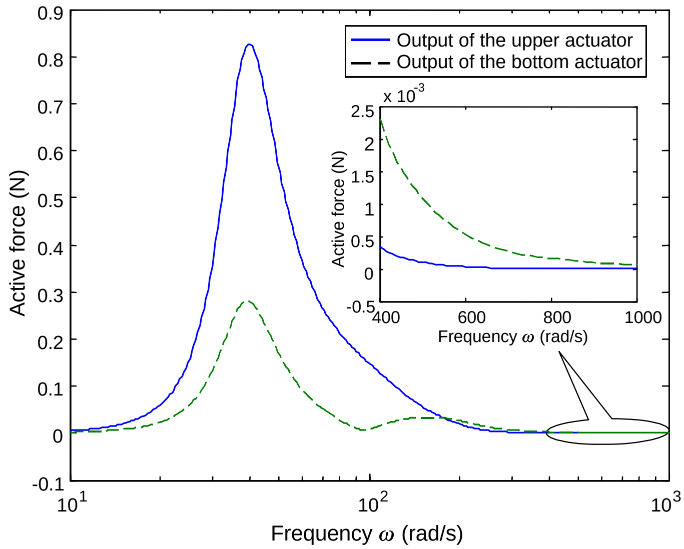

Fig. 9. Output forces of actuators of active floating raft isolation system by full control.

Fig. 7 shows the transmission of power flow to every subsystem when machine control is employed. Similarly, the power transmitted into interface 1 almost equals to that transmitted into interface 2. Correspondingly, the power transmitted into interface 3 almost agrees with that transmitted into interface 4. Comparison can be made with the plots shown in Fig. 5. However, the reduction of power through the upper mounts is much greater than that through the lower mounts. It implies that the upper actuators are capable of substantially depressing the transmission of power to the next subsystem (i.e., the intermediate raft structure).

Fig. 8 shows the transmission of power flow to subsystems when raft control is employed. The action of lower actuators in the active floating raft system by raft control makes power transmission into interface 3 to deviate from that transmitted into interface 4 at the first power flow peak and in the middle and high-frequency band above 100 rad/s. However, differing from Fig. 6, the power transmitted into interface 3 is larger than that transmitted into interface 1 because of the action of active forces.

Employing the improved power flow minimization strategy, the control forces generated by actuators of full control system subjected to excitation force $F_1 = [1, 1]e^{j\omega t}$ are shown in Fig. 9. Considering a strictly symmetric isolation system, the outputs of all upper and lower actuators are, respectively, equal in theory. Therefore, selecting one of the upper actuators to simulate the behaviors of all the upper actuators and selecting one of the lower actuators to represent all the lower actuators in the study are available in this case. It is obvious that the active forces generated by the upper actuators are larger than those generated by the lower actuators at low frequencies below 180 rad/s, particularly, they reach their maximums at about 45 rad/s which corresponds to the first natural frequency of the coupled isolation system. However, with increasing of the excitation frequency, the output of lower actuators increases gradually and may exceed the output of upper actuators at last.

5. Conclusion

In this study, an innovative active–passive analytical model of the two-stage floating raft isolation system is presented. The mobility matrices of subsystems are derived by the substructure mobility technique. The action of actuators is modeled as a pair of reactive axial forces acting at the top and bottom ends of mounts. The transmission of power flow into the foundation in the active floating raft isolation system is solved. The transmission characteristics are studied to obtain valuable and significant results.

All types of control, i.e. machine control, raft control and full control, are capable of substantially reducing transmission of power flow into the foundation. The efficiency of full control is most significant. Machine control can perform efficiently only at low frequencies and it is not efficient at all at high frequencies. Raft control is efficient to isolate vibration and to reduce noise in high-frequency band, and it also works at low frequencies although its efficiency is not as good as machine control. For low-excitation frequency, machine control is good enough to reduce structural vibration. For middle and high-excitation frequency bands, raft control can be considered. For a random disturbance in a very broad frequency band, full control should be employed.

With reference to active floating raft isolation systems by full control, the outputs of upper actuators are more significant than that of lower actuators at low frequencies. In high-frequency band, the outputs of lower actuators are slightly larger than the output of upper actuators.

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